

# Calculus 2 (for Physics)

Midterm Exam

December 4, 2025 (9:00–11:00)



university of  
 groningen

## Instructions

Please read and follow these instructions!

(Lengths are measured in meters. Time is measured in seconds.)

1) A lawn mower is connected to a pole of radius  $R$  via a rope of length  $L$ . The rope is pulled tight and tangent to the pole. The lawn mower is set to move at a constant speed  $v$  perpendicular to the rope thus keeping it taut at all times as it wraps around the pole. Ignore the thickness of the rope. Use integration to calculate the distance covered by the lawn mower.

2) Compute the curvature of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at each point  $(x, y)$ . Express your answer in terms of  $x$  and  $y$ . Here  $a, b$  are positive constants.

3) Raindrops are sliding down a roof whose shape is given by the equation  $4x^2 + y^2 + 4z^2 = 16$  ( $z \geq 0$ ). Each drop moves along a path of steepest descent due to gravity (n.b.  $\vec{F} = -mg\hat{k}$ ). Find an equation for the curves raindrops must follow depending on where they land. (You will need to solve a simple differential equation.)

4) Consider the function  $u(x, y, t) = g(r) \sin(kt)$ , where  $k$  is a constant and  $r = \sqrt{x^2 + y^2}$ . Use the Chain Rule to show that  $u$  is a solution of the wave equation  $u_{tt} = u_{xx} + u_{yy}$  if  $g$  satisfies the ODE  $g''(r) + \frac{1}{r}g'(r) + k^2g(r) = 0$ .

5) Find all stationary points of the function  $f(x, y) = xe^x - (1 + e^x) \cos y$  and determine for each stationary point whether it is a local maximum, a local minimum, or a saddle point.

6) A thin metal disk occupies the region where  $x^2 + y^2 \leq 4$  in the  $xy$ -plane. Its temperature at the point  $(x, y)$  equals  $T(x, y) = 3x^2y - y^3$  (measured in degrees Celsius). Locate the points of the highest and lowest temperatures on the disk.

- write your name and student number on the top of each sheet of writing paper!
- use the writing (lined) and scratch (blank) paper provided, raise your hand if you need more paper
- start each question on a new page
- use a pen with black or blue ink
- do not use any kind of correcting fluid or tape
- any rough work should be crossed through neatly so it can be seen
- this is a closed-book exam, you are not allowed to use the textbook or your notes or any other written material (except for the formula sheet)
- you are allowed to use a simple pocket calculator
- programmable/graphing calculators are not allowed
- your work should be clearly and logically structured
- explain your reasoning using words
- show all your calculations, an answer without any computation will not be rewarded
- each problem is worth 15 points
- you get 10 free points
- you have 2 hours to solve all 6 problems
- upon completion<sup>1</sup> submit your worksheets at the front desk

<sup>1</sup>At the end of the exam or after you finished, whichever is sooner.